

Higher Curvature Supergravity, Supersymmetry Breaking and Inflation^{*}

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dedicated to the memory of Bruno Zumino

Abstract

In these lectures, after a short introduction to cosmology, we discuss the supergravity embedding of higher curvature models of inflation. The supergravity description of such models is presented for the two different formulations of minimal supergravity.

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1 Introduction

These lectures are devoted to the application of higher curvature supergravity to a particular class of cosmological models for inflation in which the “inflaton” field is identified with the “scalaron”. The latter is a purely gravitational mode which arises when we add to the Einstein-Hilbert action a term quadratic in the (scalar) curvature

$$\mathcal{L}_{modified} = \mathcal{L}_{EH} + \alpha R^2 = \frac{1}{2\kappa^2}R + \alpha R^2, \quad (1)$$

where $\kappa^2 = 8\pi G = M_P^{-2}$. This theory is “dual” to standard Einstein gravity coupled to a scalar field. The revival of these models was motivated by the fact that the recent experiments Planck [1] and BICEP2 [2] seem to favor simple one-field cosmological models for inflation even if there is a tension between the two experiments. In fact, while for the slow-roll parameter

$$n_S = 1 - 6\epsilon + 2\eta \approx 1 - \frac{2}{N} \approx 0.96 \quad (2)$$

i.e., the spectral index of scalar perturbations, the same formula agrees, for the other slow-roll parameter r , the tensor-to-scalar ratio, a value of

$$r = \frac{12}{N^2} \quad (3)$$

as in Starobinsky inflation [3] and Higgs Inflation [4], seems to be favored by the Planck collaboration, which reports

$$r < 0.08, \quad (4)$$

whereas chaotic models [5] like the quadratic one, predicts typically

$$r = \frac{8}{N} \approx 0.2, \quad (5)$$

and are favored by BICEP2. The slow-roll parameters ϵ, η and the number of e-folds N ($\sim 50 - 60$) during inflation [6] are defined in terms of the scalar potential $V(\phi)$ of a canonically normalized inflaton field with Lagrangian

$$\mathcal{L} = \mathcal{L}_E - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \quad (6)$$

as

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V}, \quad N = \frac{1}{M_P^2} \int_{\phi_{end}}^{\phi_{init}} \frac{V}{V'} d\phi. \quad (7)$$

In the next section, we will recall some well known matter from Cosmology, and, in particular, the description of the inflaton as a component of a cosmological perfect fluid. In section 3. we

describe the Starobinsky model. In section 4. we present the supergravity embedding of the Starobinsky and chaotic inflation in the two different formulations of the $\mathcal{N} = 1$ supergravity. In addition, we show how integrating out the sgoldstino multiplet, the Volkov-Akulov-Starobinsky supergravity emerges. Finally, section 5 contains our conclusions.

2 A (short) Introduction to Cosmology

The Standard Model for Cosmology describes the Universe as made of different forms of energy densities acting as sources of the gravitational field. The latter in turn is described by a Friedmann-Lemaître-Robertson-Walker geometry (FLRW) with metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (8)$$

where the three-dimensional constant-time slice is a maximally symmetric space, the Riemann tensor of which satisfies (for constant t)

$$R_{ijkl} = \frac{k}{a^2} (g_{ik}g_{jl} - g_{il}g_{jk}), \quad k = 0, \pm 1. \quad (9)$$

For $k > 0$, $k < 0$ and $k = 0$ we refer to closed, open or flat Universe, respectively. $a(t)$ is the “scale factor” which tells us how big is the 3D slice at (comoving) time t . The above assumptions are motivated by the “Copernican Principle”, namely that our Universe looks isotropic and homogeneous. Isotropy says that space looks the same in any direction and homogeneity that the metric looks the same everywhere. If a space is isotropic everywhere, then it is homogeneous as well. If we have isotropy and homogeneity, then the 3D slice is a maximally symmetric space, i.e.,

$$a) \quad \frac{SO(4)}{SO(3)} \quad (k > 0), \quad b) \quad \frac{SO(3,1)}{SO(3)} \quad (k < 0), \quad c) \quad E_3 \quad (k = 0). \quad (10)$$

The Einstein equations are written as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}, \quad (R = R_\mu{}^\mu), \quad (11)$$

where $T_{\mu\nu}$ is the energy-momentum (stress energy) tensor, or, equivalently,

$$R_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right), \quad (12)$$

which implies

$$T = T_\mu{}^\mu = -\frac{1}{\kappa^2}R. \quad (13)$$

Note that if the energy-momentum tensor is that of a vacuum energy,

$$T_{\mu\nu} = -\Lambda g_{\mu\nu} \quad (14)$$

where $\Lambda = \text{const.}$, by the energy-momentum conservation

$$\nabla_\mu T^{\mu\nu} = 0, \quad (15)$$

we get that

$$R = 4\kappa^2 \Lambda = \text{const.} \quad (16)$$

and we have a maximally symmetric space-time, which is de Sitter (DS) ($\Lambda > 0$), anti-de Sitter (AdS) ($\Lambda < 0$) or Minkowski ($\Lambda = 0$). But what is the general form of the energy-momentum tensor $T_{\mu\nu}$? We can answer to this question if we make the picture of the Universe as being made by perfect fluids, described by an energy density ρ and a pressure p . In this case, the energy-momentum tensor can be expressed as

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu} = (\rho + p) \left(U_\mu U_\nu + \frac{1}{4}g_{\mu\nu} \right) + \frac{1}{4}(3p - \rho)g_{\mu\nu}, \quad (17)$$

where U^μ is the relativistic four-velocity vector. We may impose an equation of state

$$p = w \rho, \quad (18)$$

where $p = p(t)$, $\rho = \rho(t)$ and $w = \text{const.}$, with $|w| \leq 1$ by the dominant energy condition. Moreover, the stress tensor conservation (15) implies

$$\frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a}, \quad (19)$$

which gives that

$$\rho \propto a^{-3(1+w)}. \quad (20)$$

Note in particular that a vacuum energy $T_{\mu\nu} \propto g_{\mu\nu}$ as in Eq. (14) leads to

$$\rho + p = 0, \quad (w = -1), \quad (21)$$

while a traceless stress tensor $T_\mu{}^\mu = 0$ implies

$$3p - \rho = 0, \quad (w = \frac{1}{3}, \text{ radiation}). \quad (22)$$

The value of the scalar curvature is

$$R = -\kappa^2 T_\mu{}^\mu = \kappa^2(1 - 3w)\rho \quad (23)$$

for a single component perfect fluid, or

$$R = \kappa^2 \sum_i (1 - 3w_i)\rho_i \quad (24)$$

in the case of a multi-component perfect fluid. For $w = -1$, we have de Sitter or anti-de Sitter space-time depending on the value of ρ (DS for $\rho > 0$ and AdS for $\rho < 0$) whereas, $R \geq 0$ for $w \leq \frac{1}{3}$ and $\rho_i \geq 0$.

Because of the symmetries of the FLRW geometry, the Einstein equations now read

$$G_{00} = \kappa^2 T_{00}, \quad (25)$$

$$G_{ij} = \kappa^2 T_{ij}, \quad (26)$$

which give the equations for the scale factor

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (27)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad (k = \pm 1, 0). \quad (28)$$

A dot and a double dot denote first and second derivatives with respect to comoving time t . By introducing the Hubble parameter

$$H = \frac{\dot{a}}{a} \quad (29)$$

and noticing that

$$\frac{\ddot{a}}{a} = \dot{H} + H^2, \quad (30)$$

we can rewrite the Einstein equations as (Friedmann equations)

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (31)$$

$$\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2}. \quad (32)$$

By defining

$$\rho_k = -\frac{3k}{8\pi G} \frac{1}{a^2}, \quad w_k = -\frac{1}{3}, \quad (33)$$

Friedmann equations (31,32) are written as

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_k), \quad (34)$$

$$\dot{H} = -4\pi G(1 + w)\rho - \frac{8\pi G}{3}\rho_k, \quad (35)$$

or, in the case of a multicomponent perfect fluid

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i, \quad (36)$$

$$\dot{H} = -4\pi G \sum_i (1 + w_i)\rho_i, \quad (37)$$

(where ρ_i includes ρ_k). By dividing Eq. (31) by H^2 and defining the density parameter

$$\Omega = \frac{8\pi G}{3H^2}\rho, \quad (38)$$

we get

$$\Omega - 1 = \frac{k}{H^2 a^2}. \quad (39)$$

In addition, if we also define the critical energy density

$$\rho_{crit} = \frac{8\pi G}{3H^2}, \quad (40)$$

we have that

$$\Omega = \frac{\rho}{\rho_{crit}}. \quad (41)$$

Similarly, Eq. (36) can be compactly written as

$$\sum_i \Omega_i = 1, \quad (42)$$

where

$$\Omega_i = \frac{\rho_i}{\rho_{crit}}, \quad (43)$$

including ρ_k . The second Friedmann equation (32) can be written in terms of the “deceleration parameter”

$$q = -a \frac{\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}(\dot{H} + H^2), \quad (44)$$

as

$$q = \frac{4\pi G}{3H^2}(\rho + 3p) = \frac{4\pi G}{3H^2}(1 + 3w)\rho, \quad (45)$$

or for several components of energy densities

$$q = \frac{4\pi G}{3H^2} \sum_i (1 + 3w_i)\rho_i = \frac{1}{2} \sum_i (1 + 3w_i)\Omega_i. \quad (46)$$

Note that Ω_k does not contribute to q (since $w_k = -1/3$). As a result, Einstein equations for an FLRW geometry may be written as

$$\sum_i \Omega_i = 1, \quad q = \frac{1}{2} \sum_i (1 + 3w_i)\Omega_i. \quad (47)$$

Values for w include

$$w = 0 \quad (\text{dust-baryonic matter}), \quad (48)$$

$$w = -\frac{1}{3} \quad (\text{curvature}), \quad (49)$$

$$w = 1 \quad (\text{fast roll scalar field}). \quad (50)$$

Whenever a single term dominates, we say that the Universe is dominated by that component of (p_i, ρ_i) or $(w_i = p_i/\rho_i, \rho_i)$. For example, for $w = 0$ we have a matter dominated Universe, for $w = \frac{1}{3}$ a radiation dominated, for $w = -1$ a vacuum-energy dominated and for $w = -\frac{1}{3}$ a curvature dominated Universe. When only one component dominates, the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 \propto \frac{8\pi G}{3} a^{-3(1+w)} \quad (51)$$

can easily be integrated to give

$$a \sim \begin{cases} t^{\frac{2}{3(1+w)}}, & w \neq 1 \\ e^{Ht}, & w = -1, \quad (H = \text{const.}) \end{cases}. \quad (52)$$

A scalar field (inflaton) can be viewed as a perfect fluid component of the Universe with Lagrangian

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi), \quad (53)$$

and stress energy tensor

$$T_{\mu\nu}^\phi = -\frac{\partial\mathcal{L}_\phi}{\partial\partial_\mu\phi}\partial_\nu\phi + g_{\mu\nu}\mathcal{L}_\phi. \quad (54)$$

In components, we find for the latter

$$T_{00}^\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (55)$$

$$T_{ij}^\phi = \left(\frac{1}{2}\dot{\phi}^2 - V(\phi) \right) g_{ij}, \quad (56)$$

which gives, after comparing with the standard form of the stress tensor of a perfect fluid

$$T_{00} = \rho, \quad T_{ij} = pg_{ij}, \quad (57)$$

that the energy density and the pressure of the inflaton are

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (58)$$

Therefore, we get that

$$\rho + p = \dot{\phi}^2, \quad \rho + 3p = 2(\dot{\phi}^2 - V), \quad (59)$$

$$\rho - p = 2V, \quad T_\mu{}^\mu = \dot{\phi}^2 - 4V, \quad (60)$$

so that we have

$$p = -\rho, \quad (w = -1) \quad \text{for} \quad \dot{\phi}^2 \ll V(\phi), \quad (61)$$

$$p = \rho, \quad (w = 1) \quad \text{for} \quad \dot{\phi}^2 \gg V(\phi), \quad (62)$$

whereas, in general

$$\frac{1}{2}\dot{\phi}^2(1 - w) = (1 + w)V. \quad (63)$$

The matter equations

$$\frac{\delta \mathcal{L}_\phi}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}_\phi}{\delta \partial_\mu \phi} = 0, \quad (64)$$

together with the Einstein equations are explicitly written as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right), \quad (65)$$

$$\dot{H} - \frac{k}{a^2} = -4\pi G \dot{\phi}^2, \quad (66)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0. \quad (67)$$

Inflation claims to solve the flatness and horizon problems. The flatness problem is the explanation for having today $\Omega \sim 1$ without fine-tuning of $|\Omega - 1|$ close to zero at early times, whereas, the horizon problem is the problem of homogeneity of the observed Universe arising from seemingly non-causally connected initial regions. Inflation occurs as long as

$$\dot{\phi}^2 \ll V(\phi), \quad |\ddot{\phi}| \ll |3H\dot{\phi}|, |V_\phi|, \quad (68)$$

where the potential energy dominates the kinetic energy for sufficient period. The inflationary regime is usually parametrized by the slow-roll parameters $(\epsilon, \eta) \ll 1$ defined in Eq. (7).

Let us also note at this point that the number of e-foldings N is defined as

$$N = \int_{t_i}^{t_f} H(t) dt = \ln \frac{a_f}{a_i}, \quad (69)$$

where $t_i(t_f)$ are some initial(final) time and $a_{i,f} = a(t_{i,f})$. The definition (69) reduces to that of eq.(7) once the field equations (65-67) are used in the slow-roll approximation. Therefore we have that

$$a_f = a_i e^N = a_i \prod_m e^{N_m}, \quad \text{for} \quad N = \sum_m N_m. \quad (70)$$

In addition, for a de Sitter background with

$$H(t) = H_0 = \text{const.} \quad (71)$$

we have

$$H_0 \Delta t = N. \quad (72)$$

3 The Starobinsky Model

The Starobinsky model is the $R + R^2$ theory. It is dual (conformally equivalent) to standard gravity coupled to a scalar field [7] with a potential giving rise to inflation. Indeed, the Starobinsky Lagrangian is (in $M_P^{-2} = 8\pi G = 1$ units)

$$\mathcal{L} = -\frac{1}{2}R + \alpha R^2. \quad (73)$$

By introducing new fields σ and Λ , the Lagrangian (73) can be written equivalently as

$$\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\sigma(\Lambda - R) + \alpha\Lambda^2, \quad (74)$$

where the field σ is a Langrange multiplier which enforces the constraint $\Lambda = R$. We may write (74) as

$$\mathcal{L} = -\frac{1}{2}R(1 + \sigma) + \frac{1}{2}\sigma\Lambda + \alpha\Lambda^2, \quad (75)$$

where we observe that we have a Jordan frame function $(1 + \sigma)$. Going to the Einstein frame through the change of variables

$$g'_{\mu\nu} = g_{\mu\nu}(1 + \sigma)^{-1}, \quad (76)$$

we get that the Lagrangian (75) is written as

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{16\alpha}(1 - e^{-\sqrt{\frac{2}{3}}\phi})^2, \quad (77)$$

where the field ϕ , the “scalaron” is defined as

$$1 + \sigma = e^{\sqrt{\frac{2}{3}}\phi}. \quad (78)$$

In Fig.1 below, the scalaron potential

$$V = \frac{1}{16\alpha}(1 - e^{-\sqrt{\frac{2}{3}}\phi})^2, \quad (79)$$

has been plotted. At $V_\phi = \partial V/\partial\phi = 0$, supersymmetry is unbroken whereas during the inflationary phase (“de Sitter plateau”), we have $\partial V/\partial\phi \neq 0$ and supersymmetry is broken.

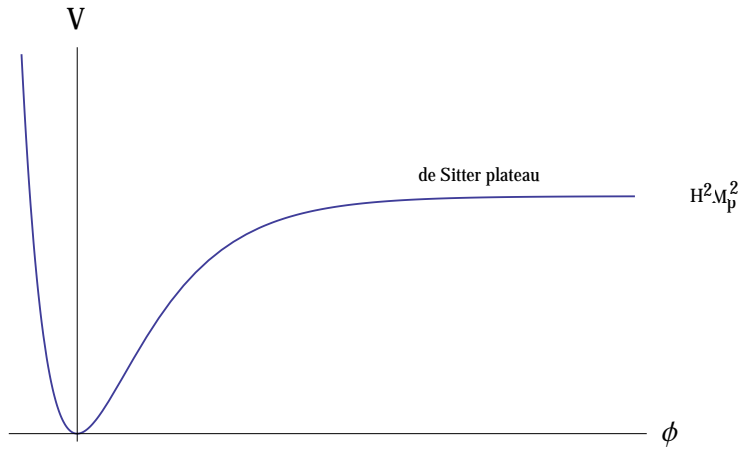


Figure 1: The “scalaron” potential in $R + R^2$ theory.

In trying to describe the Starobinsky model in supergravity, let us note that the off-shell components of the gravity field give rise to extra massive modes in higher curvature supergravity. For example let us consider a Lagrangian quadratic to curvature of the form

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_E + \mathcal{L}_{R^2} + \mathcal{L}_{Weyl^2} \\ &= \frac{1}{2\kappa^2}R + \alpha R^2 + \beta W_{\mu\nu\rho\sigma}^2.\end{aligned}$$

The graviton $g_{\mu\nu}$ has a total of six degrees of freedom, since the total number of the ten independent components of $g_{\mu\nu}$ is reduced by the number four, which is the number of diffeomorphisms

$$10 - \underbrace{4}_{\text{diffeomorphisms}} = 6. \quad (80)$$

These six degrees of freedom of $g_{\mu\nu}$ give rise to a scalar degree of freedom with mass m_0 and a massive spin-2 state with mass m_2 , according to the splitting

$$6 = \underbrace{1}_{\text{scalar}} + \underbrace{5}_{\text{spin } 2} \quad (81)$$

where

$$m_0^2 \sim \frac{1}{\kappa^2\alpha}, \quad m_2^2 \sim -\frac{1}{\kappa^2\beta}. \quad (82)$$

Therefore, the massive spin-2 state is a tachyon ($\beta > 0$) or a ghost ($\beta < 0$). This state decouples in the $\beta \rightarrow 0$ limit, leading to Starobinsky $R + R^2$ theory with dynamical degrees of freedom a massless graviton and a scalar field.

The higher curvature theory should be compared with the chaotic model [5]. The Lagrangian of the latter is

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2, \quad (83)$$

where the mass of the inflaton is $m \sim H$. It can be seen that during inflation, the de Sitter cosmological constant Λ is

$$\Lambda \sim H^2 M_P^2, \quad (84)$$

where $H/M_P \sim 10^{-5}$ for both the higher curvature and the chaotic model.

4 Supergravity Embedding

Here we present the supergravity embedding of these two models which is minimal in two respects: It uses the minimal set of multiplets needed to describe these models. It also uses the minimal

off-shell representations of the underlying local supersymmetry algebra. The latter introduces new fields which are “auxiliary” (not propagating) in the standard Einstein supergravity but become propagating when higher curvature terms are introduced. The minimal supergravity extension of such a model was derived in the late eighties [8,9] in two different forms depending of two different off-shell completion of the supergravity multiplet

$$a) \quad V_\mu^a, \quad \psi_\mu, \quad A_\mu, \quad S, \quad P, \quad (85)$$

$$b) \quad V_\mu^a, \quad \psi_\mu, \quad A_\mu, \quad b_{\mu\nu}, \quad A_\mu \rightarrow A_\mu + \partial_\mu a, \quad b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu. \quad (86)$$

The first case (a) is the old-minimal supergravity and the second case (b) is the new-minimal one. The six bosonic degrees of freedom which make the gravity multiplet to have the same number of bosons and fermions ($12b + 12f$) give two different supergravity extensions of the Starobinsky model.

The off-shell components of ($\mathcal{N} = 1$) supergravity fields give extra massive supermultiplets in higher curvature supergravity. In the old-minimal supergravity we have

$$\text{graviton } g_{\mu\nu} : \quad 6 = 1_0 + 5_2, \quad A_\mu, \quad S, \quad P : \quad 6 = 3_1 + 1_0 + 1_0 + 1_0, \quad (87)$$

$$\text{gravitino } \psi_\mu : \quad 16 - 4 = 12 = 2 \times \frac{3}{2} + 2 \times \frac{1}{2}, \quad (88)$$

which describes two massive chiral multiplet $2(\frac{1}{2}, 2(0))$ with $(4b + 4f)$ and a ghost spin-2 multiplet $(2, 2(\frac{3}{2}), 1)$. In the new-minimal supergravity, the $3 + 3$ degrees of freedom of the gauge fields $A_\mu, b_{\mu\nu}$ fill the bosonic part of a physical massive vector multiplet $(1, 2(\frac{1}{2}), 0)$ [10].

The “dual” standard supergravity action contains, in the *a*) formulation two “matter” chiral (massive) multiplets T, S ($4b + 4f$) while in the *b*) formulation contains a “massive” vector (or tensor) multiplet \mathcal{V} ($4b + 4f$). The main difference is that in the *a*) theory we are in a presence of a “four-field” model whereas in the *b*) theory we have a “single-field” inflaton model since the other three bosonic degrees of freedom combine in a massive vector.

Standard supergravity formulae allow to describe the *a*) theory in terms of a Kähler potential K and a superpotential W . It turns out that their form is

$$K = -3 \log \left(1 + T + \bar{T} - h(S, \bar{S}) \right), \quad W = \lambda T S, \quad (89)$$

where λ is a constant related to the α parameter and $h(S, \bar{S})$ is an arbitrary real function which starts with

$$h(S, \bar{S}) = S\bar{S} + \mathcal{O}(S^3). \quad (90)$$

It is possible to choose the function $h(S, \bar{S})$ appropriately in order to make the inflationary trajectory stable. Here the “inflaton” is identified with the ReT , the real part of the scalar T , while the other three scalars are “extremized”. The potential for

$$ReT = e^{-\sqrt{\frac{2}{3}}\phi} \quad (91)$$

is the Starobinsky potential. It can be shown that this theory, for any $h(S, \bar{S})$ is “dual” to a higher curvature supergravity theory. The scalar supercurvature \mathcal{R} is a chiral superfield [11]

$$\bar{D}_{\dot{a}}\mathcal{R} = 0, \quad (92)$$

and $h(S, \bar{S})$ corresponds to terms of the form $h(\mathcal{R}, \bar{\mathcal{R}})$ in the supergravity side. It is important to notice that the inflaton potential is an “F-term” potential, which means that it comes from the standard expression

$$V(T, S) = e^K \left(D_i W D_{\bar{j}} \bar{W} K^{i\bar{j}} - 3|W|^2 \right), \quad (i, j = S, T). \quad (93)$$

The inflaton potential is then

$$V(\phi) = V(T, S) \Big|_{\frac{\partial V}{\partial \bar{S}}=0, \frac{\partial V}{\partial ImT}=0}. \quad (94)$$

It happens that all supersymmetric models for the inflaton potential considered in the literature [12–24] are mostly deformations of the previous model with modification of $K(T, \bar{T}, S, \bar{S})$ and of $W(T, S)$ but still keeping the same (S, T) chiral multiplet content.

It is possible to show that at least two multiplets are needed to get an inflationary potential. In fact former theories with higher supercurvature terms of F-term type with chiral function $f(\mathcal{R})$ ($\bar{D}f = 0$) were considered in the past [25] but were shown [13, 26] not to produce an inflationary potential.

An important deformation of the (S, T) model from which the concept of “attractor” [18] came from, is a superpotential of the type

$$W(S, T) = Sf(T), \quad (95)$$

which allows bosonic potentials containing arbitrary functions of the inflaton $f(\tanh \frac{\phi}{\sqrt{6}})$. These theories are no longer equivalent to pure higher curvature supergravity but in certain cases, to

higher curvature coupled to a (single) chiral multiplet. For instance, taking K as in (89) but W now as (95), the dual higher derivative supergravity is a matter coupled theory with [27]

$$\Phi = e^{-\frac{1}{3}K} = 1 + T - \frac{f(T)}{f'(T)} + \bar{T} - \frac{\bar{f}(\bar{T})}{\bar{f}'(\bar{T})} \quad (96)$$

and a term

$$\frac{1}{|f'(T)|^2} \mathcal{R} \bar{\mathcal{R}}. \quad (97)$$

Both terms become T -independent if $f(T) = aT$.

The b) formulation gives directly a single-field inflation model where a “D-term” potential for the massive superfield is generated. The most general self-interaction of such massive vector multiplet with spin content $(1, 2(\frac{1}{2}), 0)$ resides on a real function J of a real variable C : $J(C)$ [28] The bosonic part of the supergravity action is

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) + \frac{g^2}{2}J''(C)B_\mu B^\mu + \frac{1}{2}J''(C)\partial_\mu C \partial^\mu C - \frac{g^2}{2}J'(C)^2, \quad (98)$$

so that the potential is

$$V(C) = \frac{g^2}{2}J'(C)^2. \quad (99)$$

The equation (98) actually coincides with the self-interaction of a massive vector multiplet in global supersymmetry [29]. Note also that (98) depends only on J' , J'' , so a linear term in J shifts J' by a constant but leaves J'' invariant. This constant is the so-called Fayet-Iliopoulos (FI) term. By using the Stückelberg trick one writes (98) as a gauge theory by shifting

$$A_\mu = B_\mu - \frac{1}{g}\partial_\mu a, \quad (100)$$

so that

$$\frac{g^2}{2}J''(C)B_\mu B^\mu = \frac{g^2}{2}J''(C)\left(A_\mu + \frac{1}{g}\partial_\mu a\right)^2. \quad (101)$$

In the limit $g \rightarrow 0$ the theory becomes

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) + \frac{1}{2}J''(C)\left(\partial_\mu a \partial^\mu a + \partial_\mu C \partial^\mu C\right)^2. \quad (102)$$

The (a, C) variables can be complexified to $T = -C + i\sqrt{\frac{2}{3}}a$ and the J -function can be interpreted as a Kähler potential

$$J = -\frac{1}{2}K(\text{Re}T). \quad (103)$$

The higher curvature supergravity in the b) formulation is “dual” to a self-interacting massive vector multiplet with a very precise choice [14] of

$$J(C) = \frac{3}{2} \left(\log(-C) + C \right). \quad (104)$$

Computation of the potential, for a canonically normalized field

$$C = -e^{\sqrt{\frac{2}{3}}\phi}, \quad (105)$$

leads to Starobinsky potential and Lagrangian [14, 15]

$$\mathcal{L} = \dots - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{9}{8}g^2(1 - e^{-\sqrt{\frac{2}{3}}\phi})^2, \quad (106)$$

so that the supersymmetric generalization just reproduces the single-field Starobinsky model with $\alpha \propto g^{-2}$. It is interesting to observe that the particular form of $J(C)$ corresponds to an $SU(1,1)/U(1)$ symmetric Kähler manifold with a parabolic isometry being gauged. For a Kähler potential

$$K = -3\alpha \log Re T, \quad (107)$$

the curvature is

$$R(C) = \frac{J'''(C)^2 - J''(C)J^{IV}(C)}{2J'''(C)^2} = -\frac{2}{3\alpha} \quad (108)$$

and for $\alpha \rightarrow \infty$, the curvature vanishes $R(C) \rightarrow 0$. The α -dependent potential becomes [14]

$$V(\phi) = \frac{9}{8}g^2(1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi})^2 = \frac{9}{8}g^2 P(C)^2, \quad (109)$$

where $P(C) = J'(C)$. Note that the canonical variable ϕ is related to the C variable by the equation

$$J''(C) = -\left(\frac{d\phi}{dC}\right)^2 = P'(C). \quad (110)$$

It follows then, with $P(\phi) = P(C(\phi))$, $P'(C) = dP/dC$ and $P'(\phi) = dP/d\phi$

$$P'(C) = P'(\phi) \frac{d\phi}{dC} = -\left(\frac{d\phi}{dC}\right)^2, \quad (111)$$

so that

$$P'(\phi) = -\frac{d\phi}{dC} \quad (112)$$

and

$$C(\phi) = \int d\phi \frac{dC}{d\phi} = - \int d\phi \frac{1}{P'(\phi)}, \quad (113)$$

$$J(C) = \int dC J'(C) = \int P(\phi) \frac{dC}{d\phi} d\phi = - \int \frac{P(\phi)}{P'(\phi)} d\phi. \quad (114)$$

In addition, the curvature in the ϕ -variable is

$$R(\phi) = -4 \frac{P'''(\phi)}{P'(\phi)}, \quad (115)$$

whereas, the kinetic term of the Kähler manifold is [30]

$$\frac{1}{2} J''(C) \left(\partial_\mu a \partial^\mu a + \partial_\mu C \partial^\mu C \right)^2 = -\frac{1}{2} \left[(\partial_\mu \phi)^2 + P'(\phi)^2 (\partial_\mu a)^2 \right]. \quad (116)$$

The previous equations allow us to compute $C(\phi)$ once $P'(\phi) = -d\phi/dC$ is solved.

The one-field supergravity model for inflation can be deformed in two ways:

1. Simply change $J(C)$, i.e., change the Kähler manifold.
2. Do not change the manifold but change its gauged isometry.

For the case of symmetric spaces, this procedure generates five models. Three with constant curvature depending whether a parabolic, elliptic or hyperbolic isometry is gauged and two with vanishing curvature where the parabolic or elliptic isometry is gauged [30].

4.1 Chaotic Inflation

With an “F-term” multi-field potential term, it is hard to obtain (at most in some directions of the field space) a quadratic potential. One way is to impose a shift symmetry on the Kähler potential [17, 23, 24, 31]. In terms of the (T, S) chiral fields this exchange the role of (ImT, ReT) since it is now the ImT which plays the role of the inflaton. It is then natural, in the supergravity dual to call this scenario “imaginary Starobinsky model” [20] even if a coupling to matter is needed in order to stabilize the ReT component.

In the case of chaotic inflation in the $b)$ single-field supergravity formulation, an exact model is possible since we can take a flat Kähler space where we gauge a parabolic isometry (translations). The alternative gauging of an elliptic isometry would give a quartic potential. For this case,

$$J''(C) = const., \quad J(C) = -\frac{m^2}{2} C^2 + \xi C, \quad (117)$$

but the FI term is irrelevant. Then

$$P(\phi) = \phi, \quad \text{and} \quad V(\phi) = \frac{1}{2} m^2 \phi^2. \quad (118)$$

This model can also be obtained from the constant curvature case by taking the limit [18]

$$\alpha \rightarrow \infty, \quad g^2 \rightarrow \infty, \quad \text{with} \quad m^2 \propto \frac{g^2}{\alpha} \text{ fixed}, \quad (119)$$

so that

$$g^2(1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi})^2 \rightarrow \frac{1}{2}m^2\phi^2. \quad (120)$$

4.2 Integrating out the sgoldstino multiplet: The Volkov-Akulov-Starobinsky Supergravity

We observe that the above potential is a D^2 term so during inflation D is large and the gaugino is the goldstino. The decoupling of the other (chiral) component occurs when $J''(C) \rightarrow 0$ and we get an unbroken gauge symmetry in de Sitter space (Freedman model [32]). Supergravity can be formulated in different conformal gauges (different Jordan functions). In the type a) formulation of inflation [12], there are three basic fields (chiral superfields), the conformon multiplet S_0 , which is not physical, the scalaron multiplet T , which contains the inflaton and the goldstino multiplet S which contains the sgoldstino. The latter is the goldstino partner and it is just $S|_{\theta=0}$ as supersymmetry is linearly realized. The superpotential $W = ST$ has F-terms

$$\frac{\partial W}{\partial S} = T \neq 0 \quad \text{during inflation}, \quad (121)$$

$$\frac{\partial W}{\partial T} = S = 0 \quad \text{during inflation and later}. \quad (122)$$

In fact, this explains why in the a) formulation two chiral fields are needed.

A new effective Lagrangian, since supersymmetry is badly broken during inflation, can be obtained replacing the sgoldstino multiplet S by the Volkov-Akulov superfield X which satisfies the constraint

$$X^2 = 0 \quad (123)$$

and allows to express X in terms of the goldstino G_α as [33]

$$X = \frac{G_\alpha G^\alpha}{2F_X} + \sqrt{2}\theta^\alpha G_\alpha + \theta^\alpha \theta_\alpha F_X. \quad (124)$$

In the dual supergravity theory, this corresponds to the chiral scalar supercurvature \mathcal{R} to become nilpotent [34]

$$\mathcal{R}^2 = 0. \quad (125)$$

Let us recall that the Volkov-Akulov Lagrangian is

$$\mathcal{L}_{VA} = f^2 \det V_{\alpha\mu}, \quad (126)$$

where f is the SUSY breaking parameter and

$$V_{\alpha\mu} = \delta_{\alpha\mu} + \frac{i}{f^2} \bar{G} \gamma_\alpha \partial_\mu G. \quad (127)$$

Supersymmetry is non-linearly realised and eq. (126) is invariant under the transformation

$$\delta G = f\epsilon + \frac{i}{f} \bar{G} \gamma^\mu \epsilon \partial_\mu G. \quad (128)$$

In superspace, we can write

$$\mathcal{L} = X \bar{X} \Big|_D + f X \Big|_F, \quad (X^2 = 0). \quad (129)$$

When coupled to supergravity, one gets a theory of a massive gravitino coupled to gravity with Kähler potential and superpotential

$$K = -3 \log(1 - X \bar{X}) = 3X \bar{X}, \quad W = fX + W_0. \quad (130)$$

The vacuum energy and the gravitino mass turn out to be

$$V_0 = \frac{1}{3} |f|^2 - 3|W_0|^2, \quad m_{3/2} = |W_0|, \quad (131)$$

whereas the Noether current is

$$J_{\alpha\mu} \sim f \gamma_\mu G + \dots \quad (132)$$

Now we may couple V-A to supergravity and to the scalaron multiplet. The massive spin 3/2 action is of the form

$$\frac{1}{2\kappa^2} R + \psi \partial \psi + m_{3/2} \psi \sigma \psi + \kappa^2 \psi^4 - V(f, m_{3/2}). \quad (133)$$

The constrained V-A superfield X is then coupled to the scalaron leading to an almost standard supergravity with kähler potential and superpotential

$$K = -3 \log(T + \bar{T} - X \bar{X}), \quad W = MXT + fX + W_0, \quad X^2 = 0. \quad (134)$$

The above data give rise to a potential (with no-scale structure $V \geq 0$) for the scalar T

$$V = \frac{|MT + f|^2}{3(T + \bar{T})^2}. \quad (135)$$

By defining

$$T = e^{\sqrt{\frac{2}{3}}\phi} + ia\sqrt{\frac{2}{3}}, \quad (136)$$

one gets [34]

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12}(1 - e^{-\sqrt{\frac{2}{3}}\phi})^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}(\partial a)^2 - \frac{M^2}{18}e^{-2\sqrt{\frac{2}{3}}\phi}a^2 \\ &= \mathcal{L}_{Starobinsky} + \mathcal{L}_{axion} + (\text{fermionic terms}). \end{aligned} \quad (137)$$

The dual supergravity action is

$$\mathcal{L}(S_0, \mathcal{R}) = -[S_0\bar{S}_0 - \frac{\mathcal{R}\bar{\mathcal{R}}}{M^2}]_D + (W_0 + \xi\frac{\mathcal{R}}{S_0})S_0^3 + \sigma\mathcal{R}^2S_0. \quad (138)$$

The bosonic part of this action can be obtained by dualizing the previous action (137) having set

$$e^{\sqrt{\frac{2}{3}}\phi} = 1 + 2\chi, \quad (139)$$

and Weyl rescaling

$$g_{\mu\nu} \rightarrow (1 + 2\chi)g_{\mu\nu}. \quad (140)$$

The result is the Lagrangian

$$\mathcal{L} = \frac{1}{2}(1 + 2\chi)R - \frac{1}{2}\frac{(\partial a)^2}{1 + 2\chi} - \frac{M^2}{3}(\chi^2 + \frac{a^2}{6}), \quad (141)$$

which can be written equivalently as

$$\mathcal{L} = \frac{1}{2}(1 + 2\chi)R - \frac{M^2}{18}a^2 + A^\mu\partial_\mu a + \frac{1}{2}(1 + 2\chi)A_\mu^2. \quad (142)$$

The dual Lagrangian is then obtained by integrating over a and χ . The result is ($A_\mu \rightarrow \sqrt{\frac{2}{3}}A_\mu$)

$$\mathcal{L} = \frac{1}{2}\left(R + \frac{2}{3}A_\mu^2\right) + \frac{3}{4M^2}\left(R + \frac{2}{3}A_\mu^2\right)^2 + \frac{3}{M^2}(\nabla^\mu A_\mu)^2. \quad (143)$$

This is the $R + R^2$ Lagrangian with $S = P = 0$. Note that the axion field is much heavier than ϕ during inflation where $\phi = \phi_0$ is positive and large

$$m_\phi^2 = \frac{M^2}{9}e^{-2\sqrt{\frac{2}{3}}\phi_0} \ll m_a^2 = \frac{M^2}{9}. \quad (144)$$

5 Conclusions

In these lectures, we have presented the supergravity embedding of higher curvature models of inflation. The prototype of such models is the Starobinsky $R + R^2$ gravity. This theory does not describe only the GR degrees of freedom, i.e. the helicity-2 massless graviton, but in addition it propagates a scalar degree, the scalaron. It also predicts a tiny value for the tensor-to-scalar ratio r due to an additional $1/N$ suppression with respect to the scalar tilt n_S , which is in perfect agreement with the Planck data. However, this prediction is in conflict with the BICEP2 results. This tension between Planck and BICEP2 is expected to be resolved soon. We should stress that the inflationary predictions of the Starobinsky model is identical to leading order with that of Higgs inflation [4]. As it has been shown in [35], this is due to the fact that Higgs inflation and Starobinsky model are identical during slow-roll, where the kinetic terms of the Higgs and the scalaron are subleading with respect to their potentials. We have presented how the bosonic Starobinsky model can be embedded in $\mathcal{N} = 1$ minimal supergravity. In fact, since it is a higher curvature theory, it is described both in old-minimal (formulation a)) as well as in new-minimal ((b) formulation) $\mathcal{N} = 1$ supergravity. The supergravity formulation of the chaotic model has also been presented.

Concluding, we would like to stress that the Starobinsky model and its descendants as well as their supergravity avatars cannot be excluded as long as BICEP2 results are not independently confirmed.

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